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Periodic heat transfer in a vertical plate fin cooled by a forced convective flow

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Abstract—This study investigates a vertical plate fin cooled by an oncoming turbulent flow. The base temperature of the fin varies periodically and the local heat transfer coefficient of the fin surface is determined by a conjugate convection–conduction analysis rather than arbitrarily specified. The heat conduction problem within the fin coupled with the boundary layer flow problem outside the fin surface are solved simultaneously. The calculated results include the effects of the conjugate convection–conduction parameter *Nc* as well as the oscillation amplitude and frequency of the base temperature on the fin cooling behavior.

INTRODUCTION

In the fin cooling analysis, the convective heat transfer coefficient of the fin surface is usually specified. Sparrow and Acharya [1] analyzed this kind of problem by simultaneously solving the conduction equation in the fin and the natural convective heat transfer for the cooling fluid and named the analysis the conjugate convection–conduction problem. Later, Sparrow and Chyu [2] made a similar analysis with cooling fluid changed from natural convection cooling to laminar forced convection cooling. Lien *et al.* [3] extended the analysis for a vertical fin cooled by a turbulent flow.

All of the above-mentioned studies considered the fin cooling as a steady-state heat transfer problem. However, in many practical fields, such as internal combustion engines, some special electronic components and machining processes, the heat generation is periodic instead of steady state. The base temperature of the fin is thus more likely to be a periodic oscillation rather than constant.

In this study, a conjugate convection–conduction analysis is performed for a vertical plate fin which is cooled by a turbulent forced—convection boundary layer flow. The base temperature of the fin is assumed to oscillate around a mean value.

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ANALYSIS

Consider a vertical plate fin with its base temperature T_b oscillating around a mean temperature T_m, which is greater than the ambient temperature T_∞, i.e.

$$T_b = T_m + A(T_m - T_\infty)\cos\omega t \quad A < 1 \tag{1}$$

where A is the dimensionless amplitude, ω is the frequency of oscillation, the oncoming fluid has the velocity of U_∞, and the boundary layer will be developed along the plate fin surface, see Fig. 1.

The governing equations for the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[(v+\varepsilon_m)\frac{\partial u}{\partial y}\right] \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[(\alpha+\varepsilon_h)\frac{\partial T}{\partial y}\right] \tag{4}$$

with the boundary conditions

$$u = v = 0 \quad T = T_w(x,t) \quad \text{at } y = 0 \tag{5a}$$

$$u \to u_\infty \quad T \to T_\infty \quad \text{as } y \to \infty \tag{5b}$$

where x and y denote the streamwise and normal coordinates, respectively, and u and v are the associated velocity components. T is the fluid temperature, t is the time, v and ε_m are the laminar and turbulent

NOMENCLATURE

A	amplitude of fin base temperature
A_1	amplitude of fin surface temperature
D	damping length constant
f	dimensionless stream function
h	local heat transfer coefficient
h^*	dimensionless local heat transfer coefficient
k	fluid thermal conductivity
k_f	fin thermal conductivity
L	fin length
N_1	a parameter defined by $N_1 = U_\infty L/\alpha_f$
Nc	conjugate convection–conduction parameter
Pr	Prandtl number
q	local surface heat flux
q^*	dimensionless local heat flux
Q	overall heat transfer rate
Q^*	dimensionless overall heat transfer rate
Re	Reynolds number, $U_\infty L/\nu$
t	time
T	temperature
u	velocity in the x-direction
U_∞	free stream velocity
v	velocity in the y-direction
x	streamwise coordinate
y	cross-stream coordinate.

Greek symbols

α	thermal diffusivity of fluid
α_f	thermal diffusivity of fin
δ	fin half thickness
δ_t	thermal boundary layer thickness
ε_m	eddy viscosity
ε_h	eddy diffusivity of heat
η	pseudo-similarity variable
θ	dimensionless temperature
ν	kinematic viscosity of fluid
ξ	dimensionless streamwise coordinate
τ	dimensionless time
ϕ	phase lag of fin temperature
ω	angular frequency
ω^*	dimensionless angular frequency.

Subscripts

b	quantities at the fin base
f	quantities associated with the fin
tr	condition in the transition region
w	condition at the wall
∞	quantities away from the wall.

kinematic viscosities, and α and ε_h are the laminar and turbulent thermal diffusivities.

The energy equation in the fin is assumed to be one-dimensional, i.e. the temperature difference in the y-direction is neglected because the Biot number of a thin fin is usually smaller than unity [4–6], and can be written as

$$\frac{\partial^2 T_f}{\partial x^2} - \frac{h(x,t)}{k_f \delta}(T_f - T_\infty) = \frac{1}{\alpha_f}\frac{\partial T_f}{\partial t} \tag{6}$$

with the boundary conditions

$$T_f = T_m + A(T_m - T_\infty)\cos\omega t \quad \text{at } x = L \tag{7a}$$

$$\frac{\partial T_f}{\partial x} = 0 \quad \text{at } x = 0 \tag{7b}$$

where T_f is the fin temperature, k_f is the fin thermal conductivity and $h(x, t)$ is the instantaneous local heat transfer coefficient. The coupling conditions come from the requirements that temperature and heat flux at the fin–fluid interface must be continuous, i.e.

$$T_f(x,t) = T_w(x,t) \quad \text{at } y = 0 \tag{8a}$$

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(x,t)(T_f - T_\infty) \quad \text{at } y = 0. \tag{8b}$$

By introducing the dimensionless variables of

$$\xi = x/L \quad \eta = (y/L)(Re/\xi)^{0.5} \quad \tau = \alpha_f t/L \tag{9}$$

$$f(\xi,\eta,\tau) = \psi(x,y,t)/(U_\infty x\nu)^{0.5} \tag{10}$$

$$\theta(\xi,\eta,\tau) = [T(x,y,t) - T_\infty]/(T_m - T_\infty) \tag{11}$$

$$\theta_f(\xi,\tau) = [T_f(x,t) - T_\infty]/(T_m - T_\infty) \tag{12}$$

equations (1)–(8) can be rewritten as follows:

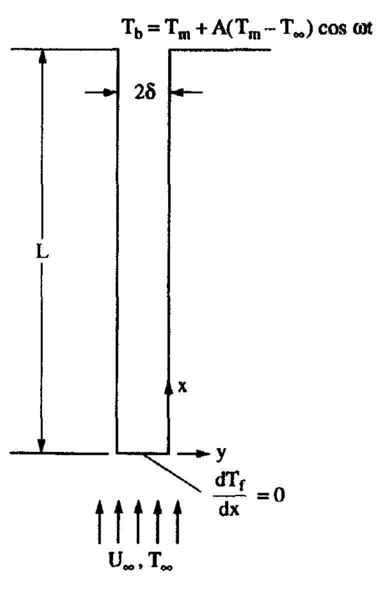

Fig. 1. Physical model and coordinate system.

for flow field:

$$\frac{\partial}{\partial \eta}\left[(1+\varepsilon_m^+)\frac{\partial^2 f}{\partial \eta^2}\right]+\tfrac{1}{2}f\frac{\partial^2 f}{\partial \eta^2}=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi \partial \eta}-\frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial \xi}\right) \tag{13}$$

$$\frac{\partial}{\partial \eta}\left[\left(\frac{1}{Pr}+\varepsilon_h^+\right)\frac{\partial \theta}{\partial \eta}\right]+\tfrac{1}{2}f\frac{\partial \theta}{\partial \eta} = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi}-\frac{\partial \theta}{\partial \eta}\frac{\partial f}{\partial \xi}+\frac{1}{N_1}\frac{\partial \theta}{\partial \tau}\right) \tag{14}$$

with boundary conditions

$$f=\frac{\partial f}{\partial \eta}=0 \quad \theta=\theta_w(\xi,\tau) \quad \text{at}\ \eta=0 \tag{15a}$$

$$\frac{\partial f}{\partial \eta}=1 \quad \theta=0 \quad \text{as}\ \eta\to\infty \tag{15b}$$

for fin:

$$\frac{\partial^2 \theta_f}{\partial \xi^2}-Nc\,h^*(\xi)\theta_f=\frac{\partial \theta_f}{\partial \tau} \tag{16}$$

with boundary conditions

$$\frac{\partial \theta_f}{\theta \xi}=0 \quad \text{at}\ \xi=0 \tag{17a}$$

$$\theta_f=1+A\cos\omega^*\tau \quad \text{at}\ \xi=1 \tag{17b}$$

conditions at the interface:

$$\theta_f(\xi,\tau)=\theta_w(\xi,\tau) \quad \text{at}\ \eta=0 \tag{18a}$$

$$h^*\theta_f\sqrt{\xi}=-\frac{\partial \theta}{\partial \eta} \quad \text{at}\ \eta=0. \tag{18b}$$

In equation (9), Re is defined as $U_\infty L/\nu$, a free stream Reynolds number, in equation (10) $\psi(x,y,t)$ is the stream function with $\partial\psi/\partial y=u$ and $\partial\psi/\partial x=-v$. The parameter N_1 in equation (14) equals $U_\infty L/\alpha_f$, and ω^* in equation (17b) is the dimensionless angular frequency defined as $\omega^*=\omega L^2/\alpha_f$. In equation (16), the dimensionless parameters Nc and h^* are defined, respectively, as

$$Nc=(kL(\sqrt{Re})/k_f\delta) \tag{19}$$

$$h^*(\xi)=-\frac{\partial \theta}{\partial \eta}(\xi,0)/[(\sqrt{\xi})\theta_f(\xi)]. \tag{20}$$

To complete the formulation for this problem, initial conditions at $\xi=0$ for the τ–η plane and at $\tau=0$ for the ξ–η plane must be specified. Since the problem is quasi-steady (periodic), the initial conditions are virtually arbitrary. In order to reduce the iteration times, the initial conditions at $\tau=0$ are given by the steady-state conditions, i.e.

$$\frac{\partial}{\partial \eta}\left[(1+\varepsilon_m^+)\frac{\partial^2 f}{\partial \eta^2}\right]+\tfrac{1}{2}f\frac{\partial^2 f}{\partial \eta^2}=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi \partial \eta}-\frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial \xi}\right) \tag{21}$$

$$\frac{\partial}{\partial \eta}\left[\left(\frac{1}{Pr}+\varepsilon_h^+\right)\frac{\partial \theta}{\partial \eta}\right]+\tfrac{1}{2}f\frac{\partial \theta}{\partial \eta}=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi}-\frac{\partial \theta}{\partial \eta}\frac{\partial f}{\partial \xi}\right) \tag{22}$$

$$\frac{\partial^2 \theta_f}{\partial \xi^2}-Nc\,h^*(\xi)\theta_f=0 \tag{23}$$

and the initial conditions at $\xi=0$ are given by setting $\xi=0$ and $\varepsilon_m^+=\varepsilon_h^+=0$ in equations (13) and (14), i.e.

$$\frac{\partial^3 f}{\partial \eta^3}+\tfrac{1}{2}f\frac{\partial^2 f}{\partial \eta^2}=0 \tag{24}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2}+\tfrac{1}{2}f\frac{\partial \theta}{\partial \eta}=0. \tag{25}$$

This means the flow is laminar at the leading edge of the fin (at $\xi=0$).

Eddy viscosity and eddy diffusivity

In the modeling of turbulence eddies, Cebeci and Smith's algebraic model (C–S model) [7] is adopted here, because this model is not only simple to use, but also very effective for the boundary layer flow considered in this study. According to the formulation of the C–S model, the turbulent boundary layer consists of inner and outer regions with separate expressions for eddy viscosity in each region. In the inner region the formulation is expressed as

$$(\varepsilon_m)_i=l^2\left|\frac{\partial u}{\partial y}\right|\gamma_{tr} \quad (\varepsilon_m)_i\leqslant(\varepsilon_m)_o \tag{26}$$

where l is the mixing length and is defined as

$$l=0.4y[1-\exp(-y/D)] \tag{27}$$

where D is a damping length constant, $D=26\nu(\tau_w/\rho)^{0.5}$.

The parameter γ_{tr} in equation (26) is an intermittency factor, which accounts for the transition between laminar and turbulent flows. γ_{tr} is shown as

$$\gamma_{tr}=1-\exp\left[-G(x-x_{tr})\int_{x_{tr}}^{x}\frac{dx}{U_\infty}\right] \tag{28}$$

where x_{tr} is the location of the starting point of transition, and the empirical factor G is given by

$$G=\frac{1}{1200}(U_\infty^3/\nu^2)Re_{x_{tr}}^{-1.34}. \tag{29}$$

In the outer region, the eddy viscosity is expressed as

$$(\varepsilon_m)_o=0.0168\left|\int_0^\infty(U_\infty-U)\,dy\right|\gamma_{tr} \quad (\varepsilon_m)_o\leqslant(\varepsilon_m)_i. \tag{30}$$

The eddy diffusivity is obtained from the turbulent Prandtl number reported by Jischa and Rieke [8], which is

$$Pr_t = \frac{\varepsilon_m}{\varepsilon_h} = a + b(Pr+1)/Pr \quad (31)$$

where $a = 0.825$ and $b = 0.0309$ for air.

Numerical procedure

The solution procedure starts with a guessed fin temperature of $\theta_f(\xi) = \cosh[\xi(1+A)]/\cosh(1.0)$ at $\tau = 0$. The boundary layer equations, equation (21) and (22), are solved by marching along the ξ-direction. The local heat transfer coefficient $h^*(\xi, 0)$ can then be determined from equation (20) after the flow solution is obtained. The calculated $h^*(\xi, 0)$ is in turn used as input to calculate the new heat conduction equation, equation (22). A newly obtained $\theta_f(\xi)$ is again imposed as the new boundary condition to repeat the calculations for flow equations. This alternative solution procedure will continue until h^* converges.

Once the solution at $\tau = 0$ is obtained, the time domain will march forward. At the new time, the same solution procedure will be applied for the flow equations [equations (13) and (14)] and for the fin equation [equation (16)]. The time marching will continue until a complete oscillation cycle is reached. In this study, a complete cycle is divided into 40 time intervals.

The calculated fin temperatures at each time interval are compared with the corresponding values obtained in the previous cycle. The solutions are finally determined whenever the maximum difference of fin temperatures is less than a given small value (say 10^{-4}). An implicit finite-difference method, namely the Keller's box method [9], was employed to perform the numerical simulation. To conserve space, the details of the solution procedure are not represented here.

RESULTS AND DISCUSSION

In carrying out the calculation, the following physical quantities are used: $Pr = 0.7$, $\nu = 1.684 \times 10^{-5}$ m^2 s^{-1}, $L = 0.08$ m, $U_\infty = 150$ m s^{-1} and $Re_{x_{tr}} = 400\,000$, such that $\xi_{tr} = 0.56$ and $N_1 = 106\,819$. The periodic heat transfer characteristics of the vertical plate fin are discussed as follows.

Instantaneous overall rate of heat transfer

In the steady-state analysis, the overall heat transfer rate from the fin can be obtained from the solution either by integrating the local convective flux at the fin surface, denoted by Q, or from the heat conducted from the fin base (at $\xi = 1$), denoted by Q_b. For the unsteady case, Q is usually not equivalent to Q_b due to the existence of the heat capacity of fin. The dimensionless forms of Q and Q_b can be expressed as

$$Q^* = \frac{Q}{k(T_m - T_\infty)Re^{0.5}} = 2\int_0^1 -\frac{1}{\xi^{0.5}}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} d\xi \quad (32)$$

and

$$Q_b^* = \frac{Q_b}{k(T_m - T_\infty)Re^{0.5}} = \frac{2}{Nc}\left.\frac{\partial\theta_f}{\partial\xi}\right|_{\xi=1}. \quad (33)$$

Figure 2 shows the calculated instantaneous Q^* and Q_b^*. Both Q^* and Q_b^* oscillate around the same mean value, which is essentially the heat transfer rate of steady-state cooling. Figure 2 also reveals there is a discordance between Q^* and Q_b^*. The existence of a discordance implies that the fin mass is either absorbing or releasing energy. Comparing Fig. 2(a) and (b), it is found that the increase of ω^* enlarges the oscillation amplitude of Q_b^* and decreases that of Q^*; the discordance between Q_b^* and Q^* is thus more obvious.

The effect of the parameter Nc is shown in Fig. 3, which indicates that the mean values (dotted lines) are greatly affected by Nc. The mean heat flux increases as Nc decreases. A low Nc means the conductivity of fin is large, which implies the heat transfer resistance of

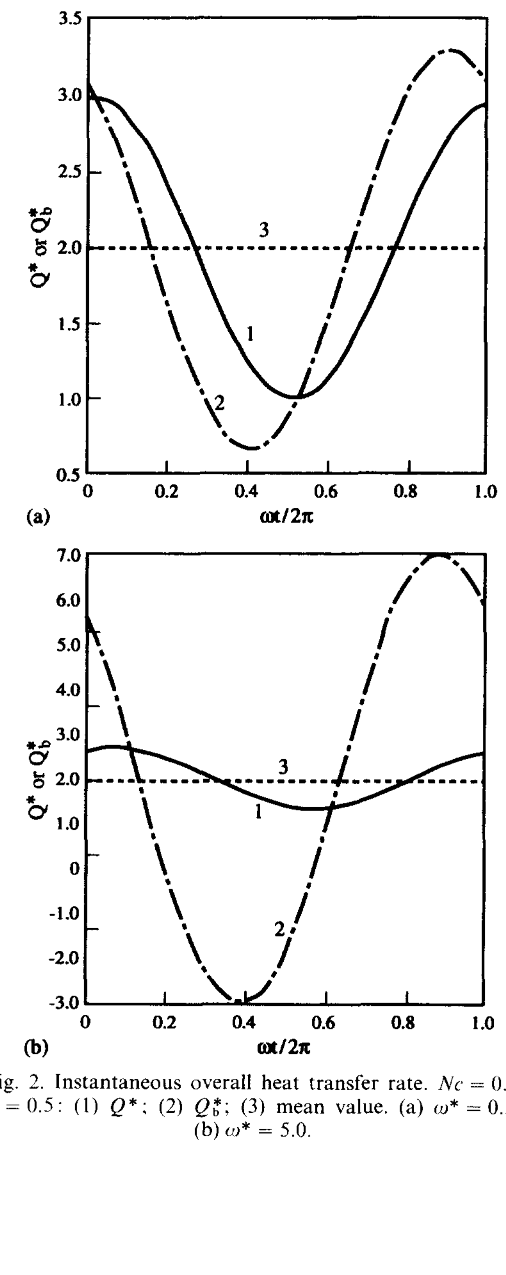

Fig. 2. Instantaneous overall heat transfer rate. $Nc = 0.5$, $A = 0.5$: (1) Q^*; (2) Q_b^*; (3) mean value. (a) $\omega^* = 0.5$; (b) $\omega^* = 5.0$.

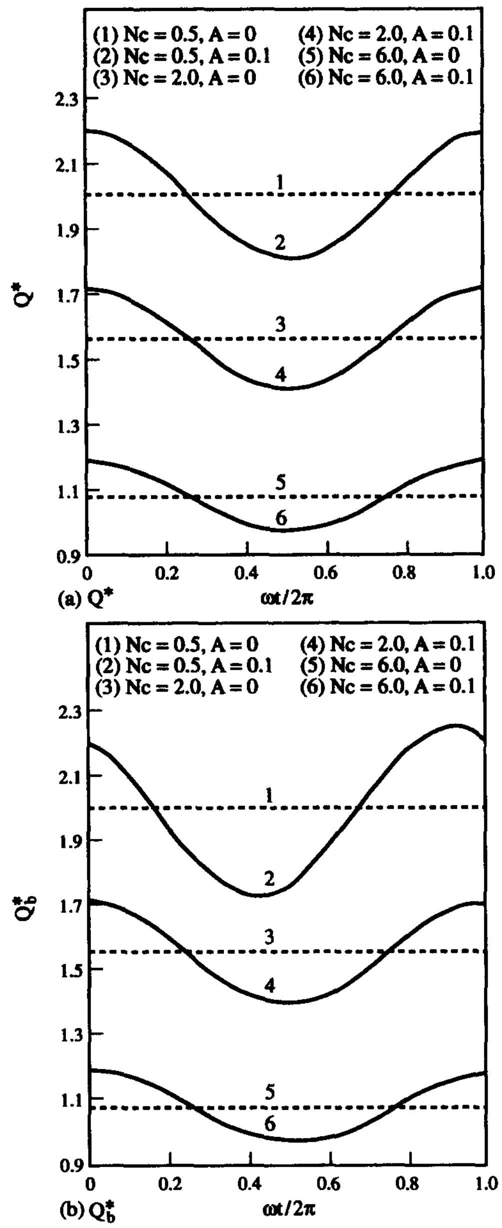

Fig. 3. The effect of Nc on Q^* and Q_b^*, $\omega^* = 0.5$. (a) Q^*; (b) Q_b^*.

the fin is small and the overall heat flux is thus larger and the amplitude of Q^* oscillation is also bigger.

From the definition $N_1 = U_\infty L/\alpha_f = (\alpha/\alpha_f)Pr\ Re$, both Pr and $\alpha/\alpha_f \approx O(1)$, but $Re \gg 1$, therefore $N_1 \gg 1$. From the flow equations [equations (13) and (14)], it can be seen that the variation of N_1 hardly affects the boundary layer solution. The physical meaning of this result is that the heating time of the boundary layer is much less than the heating time of the fin. The heating time of the boundary layer is of the order of δ_t^2/α, where δ_t is the thermal boundary layer thickness, while the heating time of the fin is of the order of L^2/α_f. The ratio of the former to the latter is of the order of $1/N_1$.

Instantaneous local heat flux

The dimensionless local heat transfer flux is expressed as

$$q^*(\xi,\tau) = \frac{qL}{k(T_m - T_\infty)Re^{0.5}} = -\frac{\partial\theta}{\partial\eta}(\xi,0,\tau)/\sqrt{\xi}. \tag{34}$$

Figure 4 reveals that q^* also varies periodically. As ξ increases, q^* decreases initially (compare curve 2 with curve 4) due to the increase of boundary layer thickness, but q^* will then increase substantially (see curves

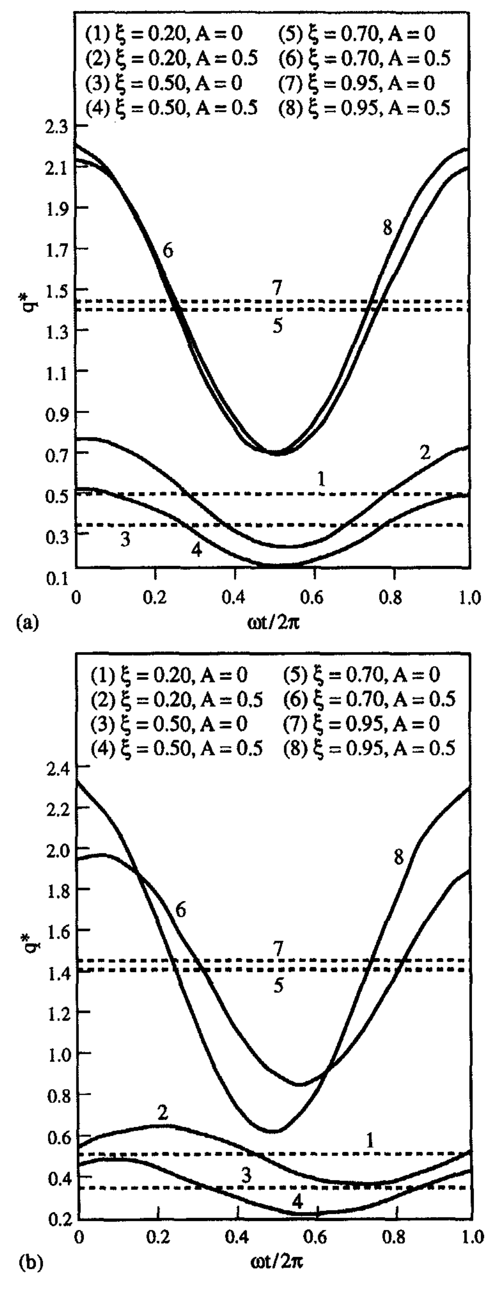

Fig. 4. The variation of q^* at different ξ and ω^* ($Nc = 0.5$). (a) $\omega^* = 0.5$; (b) $\omega^* = 5.0$.

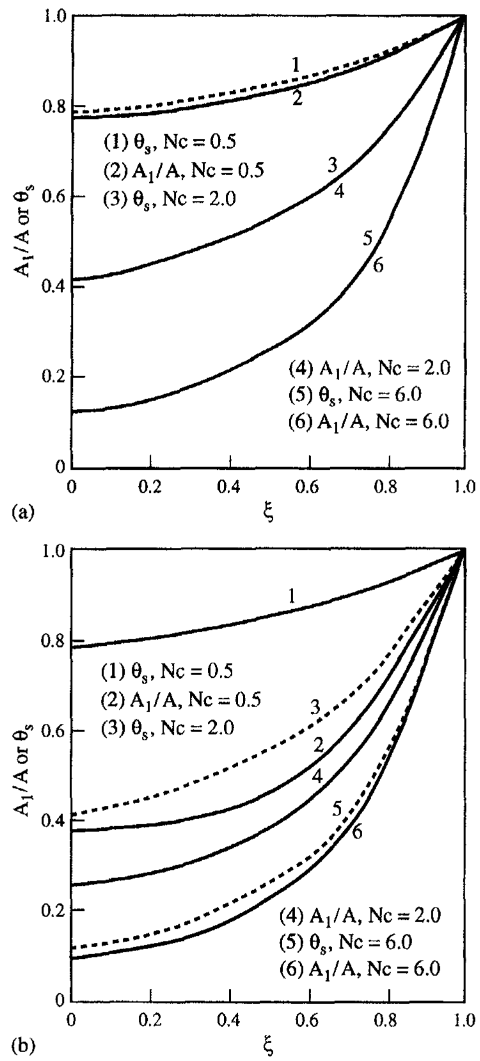

Fig. 5. θ_s and A_1/A vs ξ at different ω^*. (a) $\omega^* = 0.5$; (b) $\omega^* = 5.0$.

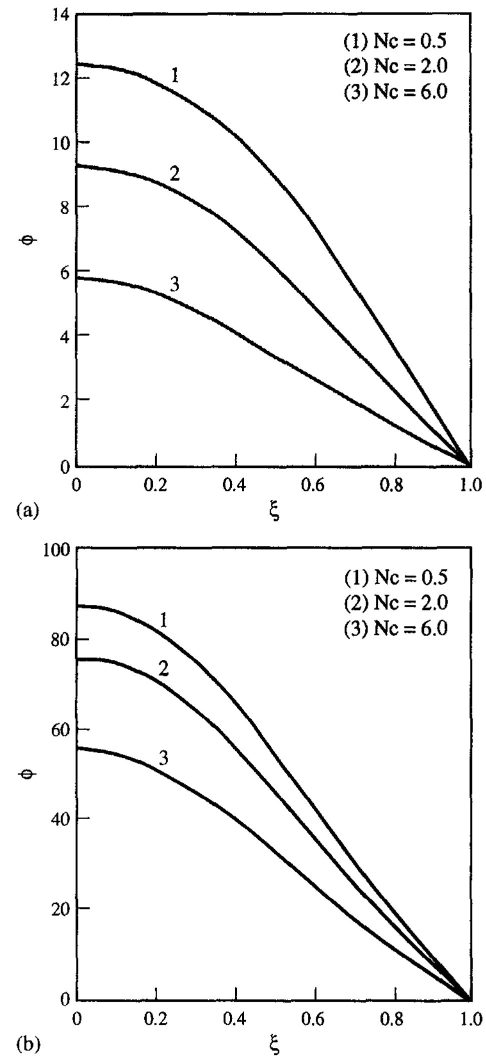

Fig. 6. ϕ vs ξ at different ω^*. (a) $\omega^* = 0.5$; (b) $\omega^* = 5.0$.

6 and 8) due to the change of flow from laminar to turbulent. Once the flow becomes turbulent the local heat flux may increase slightly with ξ; because the local Reynolds number increases with ξ, the turbulent eddies of flow may also increase to promote the local heat transfer coefficient. Comparing Fig. 4(a) and (b), it is found that the variation of q^* at different location (ξ) increases when ω^* increases.

Surface temperature of fin

The local surface temperature can be expressed by

$$\theta_f(\xi, \tau) = \theta_s(\xi) + A_1 \cos(\omega\tau - \phi) \quad (35)$$

where $\theta_s(\xi)$ is the steady-state distribution and ϕ is the phase lag behind the base temperature. As expected, θ_s and A_1/A gradually decrease, see Fig. 5, while ϕ gradually increases, see Fig. 6, as ξ approaches 0 (towards the fin tip). The above-mentioned behavior will be enhanced when Nc is smaller or when ω^* is larger. These results are consistent with the effects of Nc and ω^* on Q^* and q^*.

CONCLUSION

A numerical method has been developed to analyze a conjugate convection–conduction problem. Besides the turbulent effects, the effects due to the variations of Nc, A and ω^* on the heat transfer rate and fin temperature are also important.

Generally, the overall heat flux through the fin will increase as Nc decreases, and the variations of local heat flux and fin temperature become larger when Nc is small or when ω^* and A are large. The parameter N_1 is shown to have no significance on heat transfer results. When the value of ω^* is small, the A/A_1 curve is close to the θ_s curve and ϕ becomes small.

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